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Few-nucleon systems (theory)

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Abstract. An overview over present achievements and future challenges in the field of few-nucleon systems is presented. Special emphasis is laid on the construction of a unified approach to hadronic and electromagnetic reactions on few-nucleon systems, necessary for studying the borderline between quark-gluon and effective descriptions.

PACS. 13.40.-f Electromagnetic processes and properties – 21.45.+v Few-body systems – 25.30.-c Lepton-induced reactions – 25.20.-x Photonuclear reactions

1 Introduction

One of the most challenging topics in modern physics deals with the structure of atomic nuclei and their constituents. Despite the large efforts in the last decades, our present understanding of hadronic systems is still far from being satisfactory. The non-Abelian gauge structure of the underlying fundamental theory — quantum chromodynamics (QCD) — leads to enormous complications in practical applications. Therefore, one uses in "conventional" nuclear physics not the fundamental quarks and gluons of QCD, but nucleons, isobars and mesons as relevant degrees of freedom (d. o. f.). These so-called "effective" approaches are presently still the most promising ones for reaching a quantitative understanding of hadronic physics at low and intermediate energies below about 1 GeV excitation energy. A well-known example for the success of this effective picture is the quantitative understanding of NN-scattering data below pion threshold in terms of meson-exchange mechanisms between two interacting nucleons (for a pedagogical introduction, see [1]).

On the other side it is clear that the effective description will break down at some sufficiently high energy/momentum transfer. Moreover, it is presently not clear whether a clear cut borderline exists or whether even at relatively small energies quark and gluon degrees of freedom manifest themselves in specific reactions and observables.

It is obvious that for a detailed study of such fundamental questions a profound understanding of few-nucleon systems is inevitably necessary because the corresponding theoretical treatment is naturally the most cleanest one. Moreover light nuclei, especially the deuteron and ^{3}He, may serve as effective neutron targets so that a better understanding of few-nucleon systems may also lead to a better understanding of neutron properties. Concerning the test of effective theories, electromagnetic (e. m.) reactions have always been at the forefront in nuclear structure investigations. The electromagnetic interaction is well known from classical electrodynamics and is weak enough to allow a perturbative treatment in terms of the fine structure constant $\alpha \approx 1/137$.

In this work, selected examples of present achievements in the field of few-nucleon systems are presented. We concentrate ourselves mainly on the two-nucleon system which deserves special attention because it has the same relevance in nuclear physics as the H-atom in atomic physics. However, also some recent progress in the description of more complex few-nucleon systems is presented.

2 The two-nucleon system

2.1 Introduction

Although the two-nucleon system is the simplest few-nucleon system, it is far from being trivial. Even if we restrict ourselves to energies below the two-pion threshold, this becomes obvious by noting that quite a large number of different reactions is possible like

NN-scattering	$NN \to NN$,
Compton scattering	$\gamma d \to \gamma d$,
e. m. deuteron breakup	$\gamma d \to NN, ed \to e'NN$,
photopionproduction	$\gamma d \to \pi d, \gamma d \to \pi NN$,
elastic electron scattering	$ed \to e'd$,
Bremsstrahlung	$NN \to \gamma NN$,
pionic reactions	$\pi d \to \pi d, \pi d \rightleftharpoons NN$,
	$NN \to \pi NN$.

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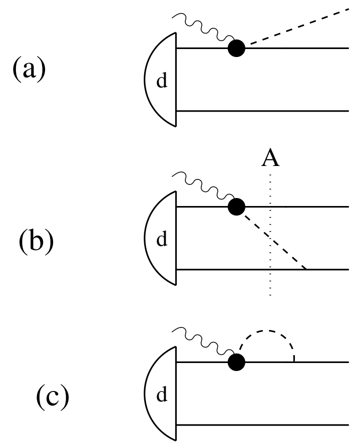

Fig. 1. Diagrammatic illustration of possible destinations of an electromagnetically produced pion. Further discussion in the text.

Thus the two-nucleon system offers a great variety of possible interaction mechanisms worthwhile to be studied. A very important point for the forthcoming discussion is the fact that these different reactions cannot be treated independently. First of all, they are linked by unitarity as becomes obvious by considering the corresponding optical theorems like

$$\mathrm{Im}\, T(NN \to NN) \sim \sigma_{tot}(NN \to NN, \pi d, \pi NN, \ldots), \quad (1)$$
$$\mathrm{Im}\, T(\pi d \to \pi d) \sim \sigma_{tot}(\pi d \to NN, \pi d, \pi NN, \ldots), \quad (2)$$
$$\mathrm{Im}\, T(\gamma d \to \gamma d) \sim \sigma_{tot}(\gamma d \to NN, \pi d, \pi NN, \ldots), \quad (3)$$

where the left sides are understood to be evaluated in forward direction. This means, for example, that the forward *Compton scattering* amplitude is related to *all* possible reactions with a photon and a deuteron in the initial state. If the restriction to energies below the two-pion threshold is dropped, of course also additional channels like 2π-, K- and η-production have to be considered.

Therefore, as a consequence of unitarity, a *unified* description of *all* possible reactions is necessary. Before we outline such an approach in some detail, let us try to understand the connection of the different above-mentioned reactions from a more intuitive point of view without referring to formal arguments based on unitarity. For that purpose, let us consider the three diagrams depicted in fig. 1. In all of them, a photon is absorbed by a deuteron producing a real or virtual pion. The three diagrams differ with respect to their final state: In diagram (a) the pion leaves the two-nucleon system, whereas in diagrams (b) and (c) it is absorbed by one of the two outgoing nucleons. Despite the close relationship of the three diagrams, their physical interpretation is completely different: diagram (a) is a contribution to photopionproduction on the deuteron, diagram (b) a part of the meson-exchange currents (MEC) to deuteron photodisintegration, and diagram (c) contributes to the anomalous magnetic moment of the hit nucleon. This simple example illustrates that single-particle properties, pion production mechanisms and meson-exchange currents are closely related. This fact again underlines the above-mentioned necessity for a unified approach to the different possible reactions in the one- and two-nucleon sector. Needless to say that such a consistent picture is in principle also required for more complex nuclei.

In a first step, we may restrict ourselves to the two-nucleon system for energies up to the Δ-region so that a basically nonrelativistic treatment should be sufficient and channels with at most one asymptotically free pion need solely to be studied. Despite these "simple" boundary conditions, the construction of such a unified approach is far from being trivial and in fact not successfully realized till now. In most existing approaches, only one or two reactions of interest are selected and the rest is just ignored. Moreover, simplifying approximations are used in order to reduce the numerical complexity. To be more precise, let us return to the different diagrams in fig. 1. In the meson-exchange contribution (b), a proper description of the propagation of the intermediate πNN-system (cut A) requires for a given invariant energy W of the system the numerical evaluation of the exact free *retarded* propagator

$$G_0(z) = (z - H_N(1) - H_N(2) - H_\pi)^{-1}, \quad z = W \pm i\epsilon, \quad (4)$$

where $H_N(i)$ and H_π describe the kinetic energy operators for nucleon "i" and the pion, respectively. Although this expression looks quite simple, its structure is quite nontrivial: It is *nonlocal* and due to its energy dependence *non-Hermitean*. Moreover, $G_0(z)$ has poles beyond pion threshold leading to logarithmic singularities known from three-body scattering theory [2,3]. Intuitively, they describe the possibility that beyond pion threshold the produced pion must not necessarily be reabsorbed by one of the nucleons but may become onshell as indicated in diagram (a). Therefore, the singularities link NN- to πNN-scattering as required by the optical theorems (1) through (3) and their correct treatment is inevitably necessary.

Due to these features of G_0, it is obvious that its numerical implementation is rather involved. Therefore, in most of the approaches an approximative treatment, the so-called *static limit* is used by assuming that the nucleons are infinitely heavy during the meson exchange (cut A in diagram (b) of fig. 1) so that in consequence no energy transfer occurs. The resulting static propagator

$$G_0^{stat} = -\frac{1}{H_\pi} \quad (5)$$

is local, energy independent and regular. Due to these nice features, which lead to large numerical simplifications, it is even nowadays very popular and used for example in state-of-the-art "high precision" NN-potentials like AV18 [4] or CD-Bonn [5,6]. The static limit works well below pion threshold but we will see that this approximation fails at higher energies. Intuitively, this is not very surprising: Due to the lack of singularities in (5), the pion is "frozen" inside the hadronic system and therefore no longer a dynamic degree of freedom.

Finally, let us make a comment on the treatment of diagram (c) in fig. 1. In conventional approaches, it is just

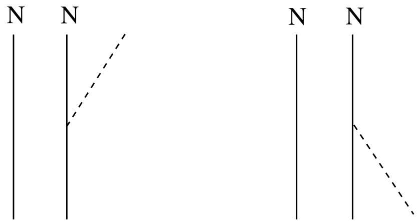

Fig. 2. Graphical illustration of meson-nucleon-nucleon vertices V_{XN} (left) and V_{NX} (right) which serve as the basic ingredients for the hadronic interaction.

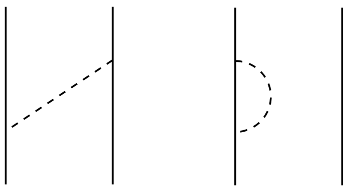

Fig. 3. Graphical illustration of the one-boson-exchange potential (left) and mesonic loop contributions to the nucleon self-energy. Both terms are generated by the second iteration of the XN-vertices depicted in fig. 1.

neglected by arguing that one uses physical nucleons as relevant effective degrees of freedom which already contain the correct anomalous magnetic moments so that the additional consideration of diagram (c) would lead to double counting. However, one has to take into account that the loop in diagram (c) is energy dependent so that its simulation by a current governed by a constant anomalous magnetic moment might be a rather crude approximation. Moreover, due to the occurrence of the retarded propagator (4), the loop can become complex so that its neglect violates the optical theorem (3) beyond pion threshold. In order to solve these problems, a careful distinction between so-called *bare* and *physical* nucleons is necessary. This conceptual complication is in most cases just circumvented by neglecting diagram (c).

2.2 Model structure

In this section, we present the general structure of our approach developed within the past years. It is suitable to study all hadronic and electromagnetic reactions on the two-nucleon system for energies up to the Δ-resonance region with at most one asymptotic free pion. In order to keep the discussion as transparent as possible, technical aspects are mostly avoided. The interested reader is referred to [7,8,9,10] concerning further details.

In order to treat a meson X as a dynamic degree of freedom, one has to work within a Hilbert space $\mathcal{H}$ where X is treated *explicitly*. Consequently, one has to allow for transitions between the NN- and the XNN-sector. In our approach, they are generated by conventional XN-vertices V_{XN} and $V_{NX} = (V_{XN})^\dagger$ $(X \in \{\pi, \rho, \omega, \sigma, \dots\})$, see fig. 2, known from NN-potential theory [1]. They serve as the basic ingredients of the hadronic interaction V in the Schrödinger equation for the deuteron bound state and the scattering equation for continuum states. The latter reads for a given invariant energy W as follows:

$$|\Psi\rangle^{(\pm)} = \pm i\epsilon G(W \pm i\epsilon)|\phi\rangle^{(PW)}, \qquad (6)$$

where $|\phi\rangle^{(PW)}$ denotes a plane-wave state (*i.e.* either a noninteracting NN-, πd- or πNN-system) and G the full propagator

$$G(z) = \frac{1}{z-H} = \frac{1}{z-H_0-V}, \qquad (7)$$

containing the potential V and the kinetic energy operator H_0. The full propagator can be rewritten in terms of the scattering amplitude

$$T(z) = V + VG_0(z)T(z) \quad \text{with} \quad G_0(z) = (z-H_0)^{-1} \qquad (8)$$

according to

$$G(z) = G_0(z) + G_0(z)T(z)G_0(z). \qquad (9)$$

It contains, therefore, the interaction V up to infinite order. The second order terms of (8), which are depicted in fig. 3, consist first of all of a one-boson-exchange potential (OBEP) of the type

$$V^{OBEP}(z) = V_{NX}(1)G_0(z)V_{XN}(2) + (1 \leftrightarrow 2), \qquad (10)$$

and a contribution to the nucleon self-energy, see equation (11) below.

Consequently, the possibility of meson production and annihilation as well as the structure of the NN-force is based on the same XN-vertex V_{XN}. This allows therefore to construct the desired unified approach. The price we have to pay is at least twofold. First of all, the NN-interaction is more complex as conventional ones because it has to be treated in the exact retarded, energy dependent manner. In our explicit realization, we use the parametrization of the Elster potential [11], which just consists of the diagrams of fig. 3 with inclusion of π-, ρ-, ω-, σ-, δ- and η-exchange. The free parameters of the corresponding vertices (cutoffs, coupling constants) are fitted to the NN-scattering phase shifts for energies up to the pion threshold.

As a second complication, the mesonic loop diagrams

$$V^{self}(z) = V_{NX}(1)G_0(z)V_{XN}(1) + (1 \leftrightarrow 2) \qquad (11)$$

appear, depicted on the right-hand side of fig. 3. In order to avoid any double counting, we have to distinguish therefore *bare* from *physical* nucleons. Whereas the first ones are the basic d. o. f. of our Hilbert space, the latter contain, among other things, the loop contributions (11). This distinction requires a proper renormalization procedure in order to formulate the model in a self-consistent manner, see [8] for more details. Its neglect leads to a severe violation of unitarity beyond pion threshold [11].

Next, we introduce the electromagnetic interaction. It is done by using the canonical gauge invariance preserving method of minimal substitution, and typical prototypes of resulting currents are depicted in fig. 4. More

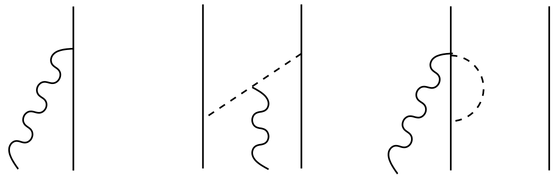

Fig. 4. Examples for current contributions in the one- and two-nucleon sector: left: one-body current; middle: meson-exchange current (MEC); right: electromagnetic loop correction.

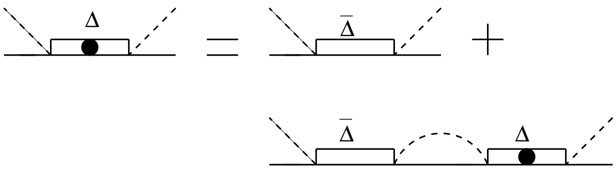

Fig. 5. πN-scattering in the P_{33} channel.

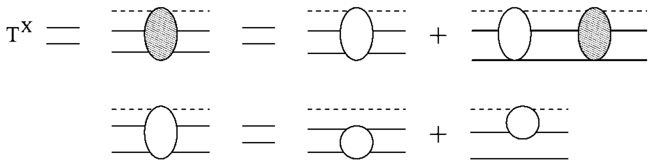

Fig. 6. Diagrammatic representation of the amplitude T^X (12).

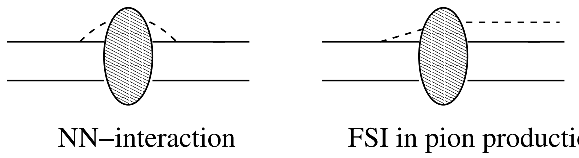

Fig. 7. The amplitude T^X (12) contributes both to the NN-interaction as well as to final state interactions (FSI) in pion production.

details can be found in [9,10]. In practice, gauge invariance (as well as unitarity) is not exactly fulfilled due to some technical reasons: Whereas the Elster potential is treated in a completely relativistic manner concerning the vertices, the vertex structure in the corresponding MEC is presently treated only nonrelativistically within a p/M_N-expansion. Moreover, MEC of at least fourth order in the πN-coupling constant are necessary to preserve gauge invariance exactly [9]. Their handling is technically very complicated and, therefore, presently neglected. These violations of gauge invariance and unitarity occur fortunately only at higher order in the $1/M_N$-expansion.

The model discussed so far is only suitable for energies below the pion threshold. In order to allow for higher energies, nuclear resonances must necessarily be incorporated. In the present approach we restrict ourselves to the Δ which is again considered as a "bare" particle ($\bar{\Delta}$) with vanishing decay width. Similar to [12], its coupling to the πN-system is generated by a suitable $\pi N\Delta$-vertex $V_{\Delta\pi}$ whose parametrization is fixed by studying πN-scattering in the P_{33}-channel, see fig. 5. In a similar manner, the electromagnetic transition $\gamma N \to \Delta$ is fixed once for all by considering photopionproduction on the nucleon in the $M_{1+}(3/2)$-multipole [9]. As next step, the Δ has to be introduced in the two-nucleon system. This is performed *nonperturbatively* within a NN-$N\Delta$ coupled-channel approach, see [8] for more details.

Last but not least, in addition the possibility of mutual interactions within the πNN-system needs to be considered. This can be tackled using standard three-body techniques for the relevant amplitude T^X depicted in fig. 6.

Neglecting three-body forces, T^X has the form

$$T^X(z) = \left(V^\pi + V^N\right) + \left(V^\pi + V^N\right) G_0(z) T^X(z), \quad (12)$$

where V^π describes the NN-interaction in the presence of a spectator pion, and V^N the πN-interaction in the presence of a spectator nucleon. As indicated in fig. 7, $T^X(z)$ contributes simultaneously to the NN-scattering amplitude as well as to final state interactions (FSI) in pion production processes.

For technical reasons we intend to parametrize these interactions in terms of suitable separable realizations [13,14]. In the present realization, solely the so-called πd-channel, *i.e.* V^π in the ${}^3S_1/{}^3D_1$ NN-channel is considered [8]. A more complete treatment of $T^X(z)$ is under construction. Moreover, the approach discussed so far is presently only realized for NN-scattering [8] and electromagnetic deuteron breakup [9,10,15]. An extension to photopionproduction as well as elastic πd-scattering will be available soon.

Our proposed model is definitely a very promising one for studying simultaneously all possible hadronic and electromagnetic reactions up to the Δ-region with at most one asymptotic free pion. Concerning alternatives to our approach, we only mention here the presently most popular one, namely effective field theory (EFT) which is based on the spontaneously broken chiral symmetry of QCD. EFT starts from the most general effective Lagrangian which is consistent with the symmetries of QCD and therefore more involved than the Lagrangians used in our approach. On the other hand one has to recognize that our approach is *nonperturbative* whereas EFT performs a simultaneous expansion in small external momenta and quark masses. It is therefore a perturbative treatment in terms of an expansion parameter Q/Λ with $Q \sim m_\pi$ and $\Lambda \sim 1\,\text{GeV}$, the chiral symmetry breaking scale. In contrast to our approach, it is presently applicable only in quite a small energy domain like NN-scattering up to pion threshold (see [16,17] and references therein), low momentum elastic electron deuteron scattering [18] or electropionproduction near threshold [19].

2.3 Deuteron breakup in the Δ-region

Next, we turn to the results of our approach for a selected choice of reactions, starting with deuteron photodisintegration. Despite its simplicity, this reaction has posed severe problems for theoreticians until the middle of the 90s. This becomes obvious from fig. 8, where experimental data for the total cross section in the Δ-region

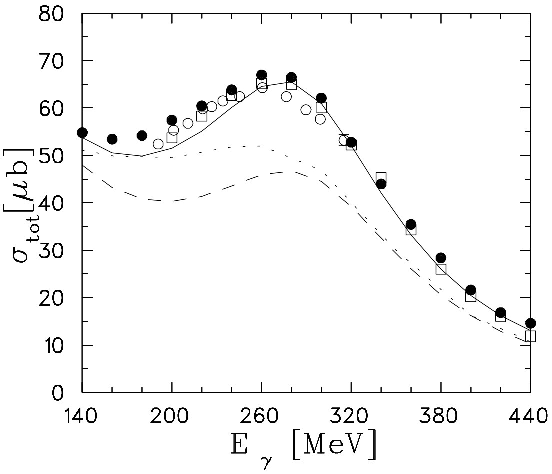

Fig. 8. The total cross section σ_{tot} of deuteron photodisintegration as a function of the photon energy. Results from Tanabe and Ohta [20] (dotted), Wilhelm and Arenhövel [21] (dashed) and Schwamb and Arenhövel [9,10] (full). Experimental data from [22] (□), [23] (○) and [24] (●).

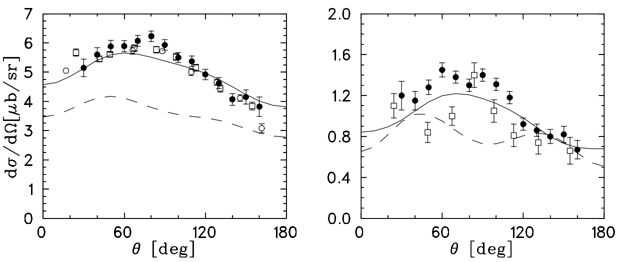

Fig. 9. Differential cross sections of deuteron photodisintegration in the center-of-mass frame for a laboratory photon energy of 260 MeV (left) and 440 MeV (right). Notation of the curves as in fig. 8.

is compared with the most sophisticated models available at that time, namely the unitary three-body approach of Tanabe and Ohta [20] as well as the model of Wilhelm and Arenhövel [21]. Similar to our treatment, realistic NN-interactions are used and a dynamical treatment of the Δ is incorporated. Moreover, a considerable conceptual improvement in comparison to earlier work was the fact that *no* free parameters occur in the photodisintegration channel because — similar as in our approach — all of them have been fixed in advance by considering πN- and NN-scattering as well as photopionproduction on the nucleon. From fig. 8 it becomes obvious that the theory clearly fails in describing the data. The predicted total cross sections are too small and a dip structure around 90° occurs in the differential cross section at higher energies which is not present in the data, see fig. 9. These problems were very severe ones because deuteron photodisintegration is the simplest photonuclear reaction on a nucleus.

In the past decade, we have made considerable efforts to solve this problem [7,9,10] reaching now an almost quantitative description of the total cross section in the Δ-region, see fig. 8. Furthermore, also the description of the differential cross section is considerably improved. This success turned out to be the combined result of various independent improvements compared to our starting point [21]. Apart from the additional incorporation of dissociation currents, the πd-channel and conceptual improvements in the description of the $\gamma N \to \Delta$-transition, retardation effects both in the hadronic interaction as well as in the MEC turn out to be very important. The latter have been partially neglected in [20,21] by using the static Paris and Bonn-OBEPR potentials, respectively, and corresponding static MEC. This result clearly indicates that even in breakup reactions of nuclei, where no asymptotic free pions occur, the latter must be treated in a dynamic manner for energies beyond pion threshold.

In a recent extension of this work, we have studied the role of retardation in deuteron electrodisintegration [15]. Neglecting polarization effects the differential cross section for this reaction in the one-photon-exchange approximation is determined by four structure functions, two diagonal ones f_L and f_T and two interference ones f_{LT} and f_{TT} [25,26]. They are functions of the squared three-momentum transfer q^2, the final state kinetic energy $E_{np} = W - 2M_N$, and the angle θ between $\boldsymbol{q}$ and the proton momentum in the final neutron-proton center-of-mass system. It turns out that retardation leads to dramatic changes in the structure functions f_L and f_{LT} for excitation energies beyond the pion threshold whereas the other structure functions f_T and f_{TT} are much less affected. This is illustrated in fig. 10 for a suitable kinematics in the Δ-region which has been studied at NIKHEF [27]. It turns out that especially the recoil charge contribution (right panel in fig. 10) is very important. This mechanism is not present in conventional static approaches due to an implicitly applied wave function renormalization procedure [28,29] whose aim is to construct orthonormalized baryonic wave functions. This concept breaks down beyond pion threshold, where the pion can become onshell and must be necessarily included in the hadronic wave functions. This fact, already discussed in [9], clearly indicates that a static treatment is only a poor approximation in reactions on the deuteron beyond pion threshold. It would of course be very important to perform experimental checks of these predictions.

2.4 The deuteron as effective neutron target

The precise knowledge of elementary particle properties is very important for a better understanding of their internal structure. With respect to the neutron as one of the most important particles, its finite lifetime forces us to consider few nucleon systems like the deuteron or ^{3}He as alternative "effective" neutron targets. The basic question is, whether for a specific neutron property of interest a specific reaction on the deuteron exists where the neutron contribution is dominant and nuclear background effects from Fermi motion, MEC, FSI, etc. are small or at least under control.

As a first example, let us consider the neutron form factors G_{En} and G_{Mn}. The magnetic form factor G_{Mn}

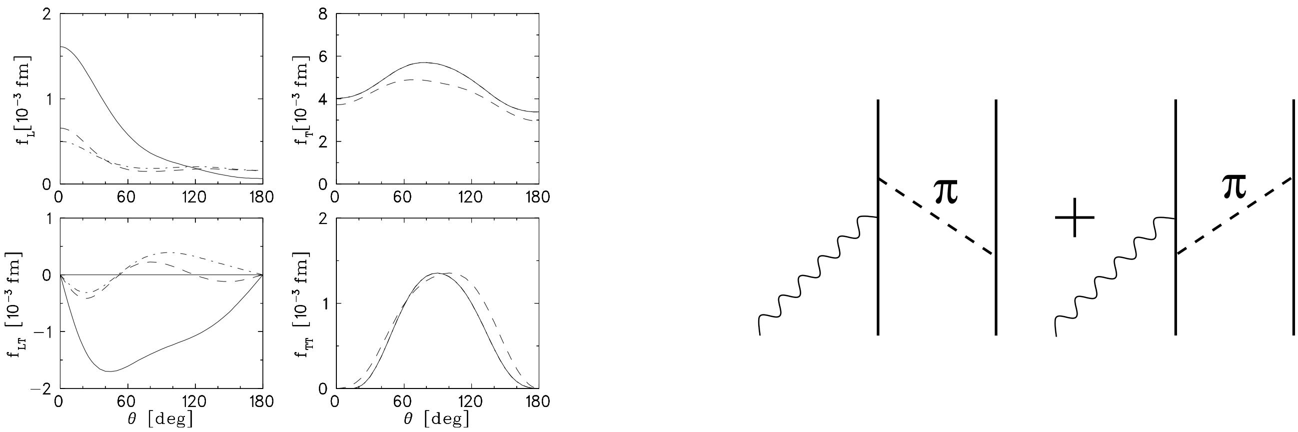

Fig. 10. Results for deuteron electrodisintegration taken from [15]. Left panel: the structure functions f_L, f_T, f_{LT} and f_{TT} for the kinematics of the NIKHEF experiment [27], *i.e.* $E_{np} = 280\,\text{MeV}$, $q^2 = 2.47\,\text{fm}^{-2}$. Notation of the curves: dashed: static approach; full: retarded approach. The additional dash-dotted curves represent the results of the retarded approach where the Coulomb monopole contribution of the recoil charge operator, depicted on the right, is switched off.

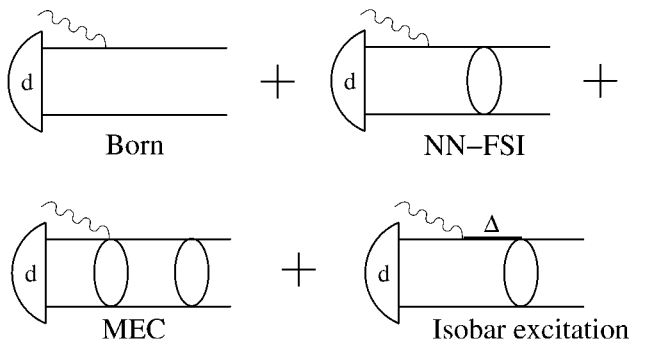

Fig. 11. Relevant diagrams in deuteron electrodisintegration.

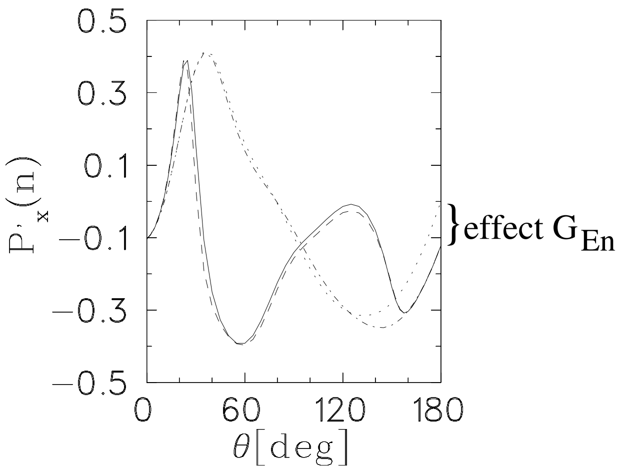

Fig. 12. The polarization P_x' of the outgoing neutron in the scattering plane perpendicular to the photon momentum as a function of the proton scattering angle θ for a squared photon four-momentum of $Q^2 = 1\,\text{GeV}^2$, a squared three-momentum transfer $q^2 = 25.67\,\text{fm}^{-2}$ and a kinetic energy of the outgoing nucleons of $E_{np} = 250\,\text{MeV}$. The neutron scattering angle θ_n is given by $\theta_n = 180° - \theta$. Notation of the curves: dotted: PWBA with $G_{En} = 0$; dash-dotted: PWBA with $G_{En} \neq 0$; dashed: full static calculation based on Bonn-OBEPR potential ($G_{En} \neq 0$); solid: full retarded calculation based on Elster potential ($G_{En} \neq 0$). In quasi-free neutron kinematics ($\theta = 180°$), one readily recognizes the sensitivity of P_x' to G_{En} as well as its insensitivity to nuclear structure effects like FSI, MEC and resonance contributions.

of the neutron can be determined from electron backscattering off the deuteron in quasi-free neutron kinematics. In this specific kinematics, the momentum of the virtual photon in the laboratory frame is completely transferred to the neutron whereas the spectator proton is at rest in the final state. These conditions lead to the rule of thumb $E_{np}/\text{MeV} = 10\,q^2/\text{fm}^{-2}$.

Compared to G_{Mn}, the electric neutron form factor G_{En} is more difficult to measure. Various possibilities to measure G_{En} have been discussed in [30]. It turned out that the cleanest determination is obtained in double polarization observables in deuteron electrodisintegration, *i.e.* $d(\vec{e}, e'\ \vec{n})p$ or $\vec{d}\ (\vec{e}, e'n)p$. The relevant diagrams contributing to this reaction are depicted in fig. 11.

In the following, we restrict ourselves to the reaction $d(\vec{e}, e'\ \vec{n})p$. In the Born approximation (PWBA), *i.e.* neglecting FSI, MEC as well as isobars, and neglecting in addition the D state of the deuteron, it turns out that in quasi-free neutron kinematics the polarization component P_x' in the scattering plane perpendicular to the photon momentum is directly proportional to G_{En} so that one has a linear relation between the observable and the quantity of interest:

$$P_x' \sim G_{En}\, G_{Mn}. \tag{13}$$

Moreover, it turns out that in quasi-free neutron kinematics the role of the background effects of FSI, MEC and isobars is under control and almost model independent, see fig. 12 as an illustrative example. This allows therefore a very clean interpretation of the existing data (consider [31] and the references therein), so that we can conclude that the deuteron is a very efficient effective neutron target with respect to the extraction of G_{En}.

We now turn to a second example where the use of the deuteron as an effective neutron target would be highly desirable. It deals with the investigation of the Gerasimov-Drell-Hearn sum rule (GDH) for various hadronic targets [32,33]. This sum rule links the anomalous magnetic moment of a particle to the energy weighted integral over the spin asymmetry of the absorption cross section. In detail it reads for a particle of mass M, charge eQ, anomalous magnetic moment κ and spin S

$$I^{GDH} = \int_0^\infty \frac{\mathrm{d}\omega'}{\omega'} \left(\sigma^P(\omega') - \sigma^A(\omega')\right) = 4\pi^2\kappa^2 \frac{e^2}{M^2} S, \tag{14}$$

where $\sigma^{P/A}(\omega')$ denote for a given photon momentum ω' the total absorption cross sections for circularly polarized photons on a target with spin parallel (P) and antiparallel (A) to the photon spin. This sum rule gives therefore a very interesting relation between a ground state property (κ) of a particle and its whole excitation spectrum. Apart from the general assumption that the integral in (14) converges, its derivation is based solely on first principles like Lorentz and gauge invariance, unitarity, crossing symmetry and causality of the Compton scattering amplitude of a particle. Consequently, a check for various targets, both from the experimental as well as from the theoretical point of view, would be very important.

Inserting the known anomalous magnetic moments of proton and neutron into (14), one obtains quite large GDH sum rule values, *i.e.* $I_p^{GDH} = 204.8\,\mu$b for the proton and $I_n^{GDH} = 233.2\,\mu$b for the neutron. On the other side, the deuteron has a small anomalous magnetic moment $\kappa_d = -0.143$ n.m. resulting in a very small GDH sum rule value of $I_p^{GDH} = 0.65\,\mu$b.

Whereas GDH measurements on proton targets can be directly performed (consider [34] and references therein), no free neutron target exists and one may try to extract I_n^{GDH} from deuteron measurements. In contrast to the extraction of G_{En}, this task is however much more complicated. First of all, let us recall that for the extraction of the electric neutron form factor *one* specific reaction (*e.g.* deuteron electrodisintegration) in *one* specific kinematics (the quasi-free one) is sufficient. On the other hand, concerning the GDH sum rule one has to determine total inclusive cross sections, *i.e.* contributions in *all* possible kinematics from very different reactions like

$$\gamma N \to \pi N, \pi\pi N, \eta N, \ \ldots \tag{15}$$

for the nucleon, and

$$\gamma d \to NN, \pi NN, \pi d, \pi\pi NN, \eta NN, \ \ldots \tag{16}$$

for the deuteron have to taken into account.

These complications become even more serious if one considers the sum of the proton and neutron value compared to the deuteron value. If one assumes that the meson production on the deuteron is dominated by the quasi-free production on the nucleons bound in the deuteron, one would expect that I_d^{GDH} should be roughly $I_p^{GDH} + I_n^{GDH}$. This assumption is however wrong by more than two orders of magnitude. Consequently, concerning the GDH

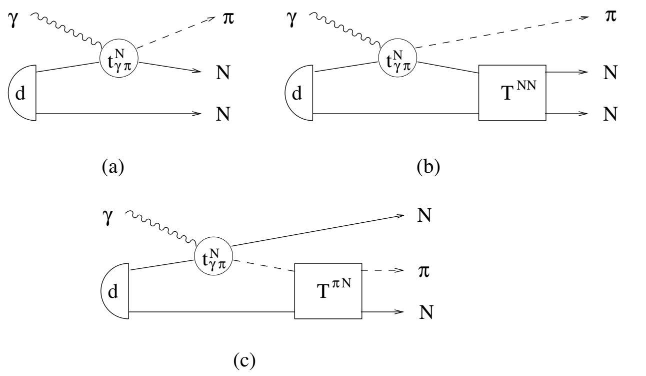

Fig. 13. Considered diagrams for single pion production. (a) impulse approximation (IA), (b) incorporation of NN-final state interaction (NN-FSI), (c) incorporation of πN-final state interaction (πN-FSI).

sum rule the deuteron reaction cannot be considered just as an incoherent sum of the proton and the neutron reaction. In order to obtain the small deuteron GDH value, strong anticorrelation effects between the different possible channels for the deuteron must occur which are not present in the elementary case. This cancellation is a challenge for any theoretical framework since it requires the above-mentioned unified consistent treatment of hadronic and electromagnetic properties for the different possible channels in a wide energy region.

In the past years, considerable efforts have been undertaken in order to obtain a more quantitative understanding of the GDH sum rule on the deuteron [35,36,37]. In the presently most sophisticated approach [37], besides deuteron photodisintegration also coherent and incoherent single and double pion production as well as η-production are considered. At the moment, the aforementioned retarded approach is only available for the breakup channel. Concerning incoherent single pion production, the considered mechanisms in our present realization are depicted in fig. 13. For the elementary production operator, the MAID model [38] is used, allowing one to extend the calculation up to photon energies of 1.5 GeV. Moreover, final state interactions are perturbatively taken into account up to the first order in the corresponding πN- and NN-scattering amplitudes. For coherent pion production, the model of [39] is used taking into account pion rescattering by solving a system of coupled equations for the NN-, $N\Delta$- and $NN\pi$-channels. It is partially similar to our approach discussed in section 2.2. However, no retardation concerning the NN-interaction and the corresponding MEC is presently taken into account. For double-pion production the evaluation is based on a traditional effective Lagrangian approach similar to the one in [40]. It is presented in great detail in [41].

Although this treatment of the GDH sum rule on the deuteron is presently the most sophisticated one, we are aware of specific shortcomings. The most serious one is the use of different approaches for the different reactions. In order to obtain a more unified picture, work is in progress to adopt the discussed retarded approach not only to the

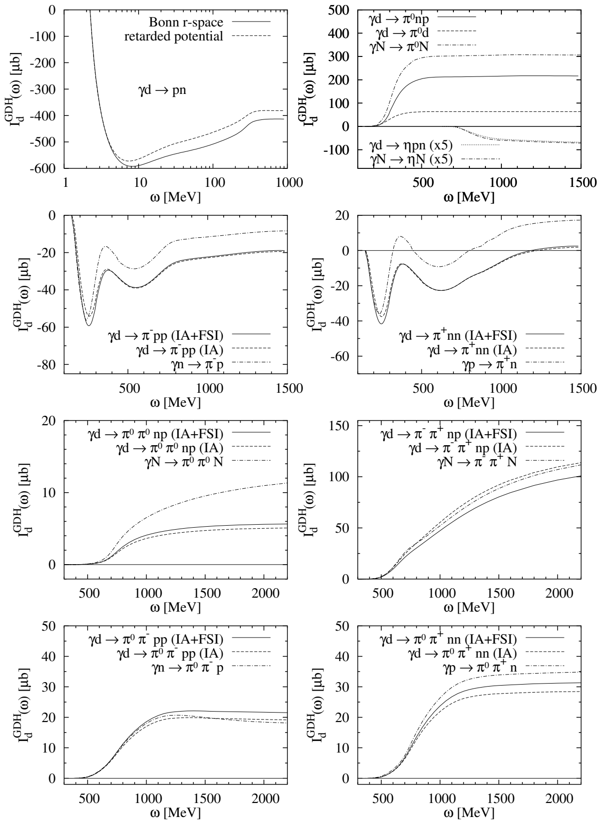

Fig. 14. Contributions of various channels to the finite GDH integral (17) as a function of the upper integration limit for deuteron disintegration, single- and double-pion and η-production on nucleon and deuteron. For the neutral charge channels π^0, η, $\pi^0\pi^0$, and $\pi^-\pi^+$, the nucleon integrals are the sum of proton and neutron integrals. See [37] for more details.

breakup channel, but at least also to single-pion production.

In order to present the results in a transparent way, we introduce for convenience the finite GDH integral as defined by

$$I^{GDH}(\omega) = \int_0^{\omega} \frac{d\omega'}{\omega'} \left(\sigma^P(\omega') - \sigma^A(\omega')\right), \qquad (17)$$

for which the results for photodisintegration, single and double pion and η-production are exhibited in fig. 14. With respect to the photodisintegration channel, at very low energies a very large negative contribution arises from the $M1$ transition to the resonant ${}^1S_0$ state which can only be reached if the spins of photon and deuteron are antiparallel. Sizeable differences especially in the Δ-region occur between our retarded approach and an older static evaluation [35] which was based on the Bonn-OBEPR potential.

Concerning single pion production, we show in fig. 14 the results both in IA and with inclusion of final state interactions (labeled as IA+FSI) together with the corresponding results for the elementary reactions. One notes besides a positive contribution from the Δ-resonance another one above a photon energy of about 600 MeV from $D_{13}(1520)$ and $F_{15}(1680)$. For charged pion production FSI effects are in general quite small. The same is true also for η-production. But FSI is nonnegligible for incoherent neutral pion production due to the non-orthogonality of the final state wave in IA to the deuteron bound state wave function, see [42] for more details. Please note moreover the significant differences between the deuteron and the corresponding nucleon values for $I^{GDH}(\omega)$. This feature occurs also in double-pion production where the largest contribution is coming from the $\pi^-\pi^+$-channel. Here the inclusion of FSI, where only NN-rescattering is presently taken into account, is quite small.

The contributions of various channels to the finite GDH integral (17) for nucleon and deuteron are listed in table 1. While for the neutron the total sum is about 8 % lower than the sum rule value, it is too large by about 28 % for the proton. Concerning the deuteron, each of the different channels (apart from η-production) produces very large contributions. Due to the large cancellation of the photodisintegration and the meson production channels, the sum of all contributions is quite small (27.31 μb). This is still somewhat too large compared to the theoretical value of 0.65 μb. However, one should keep in mind that our approach still needs to be improved due to several shortcomings as indicated above.

The strong cancellation between the regions at low and high energies is a fascinating feature clearly demonstrating the decisive role of the pion as a manifestation of chiral symmetry governing strong interaction dynamics in these two different energy regions. With respect to meson production channels on nucleon and deuteron, the different behaviour of the corresponding spin asymmetries indicates that a direct experimental access to the neutron spin asymmetry from a deuteron measurement by subtracting the one of the free proton is not possible. On the other hand, the measurement of the spin asymmetry for the different channels on the deuteron presents itself a stringent test of our present theoretical understanding of two-nucleon physics. Therefore, the experimental program at facilities like MAMI and ELSA concerning the GDH sum rule on the deuteron is very important for further progress in that field.

3 More complex few-nucleon systems

Till now, we have concentrated ourselves solely on the two-nucleon system. The present situation in the three-nucleon system is outlined in great detail in [43] and therefore not discussed here. Concerning even more complex few-nucleon systems, we want to present here merely some recent highlights obtained with the Lorentz integral transform method (LIT) [44]. The basic question in this context is, up to which mass number A and energy/momentum transfer precise *microscopic* calculations for the electromagnetic response can be performed. The most fundamental observable in this field is definitely the total inclusive cross section σ_{tot}. In conventional scattering theory, an economic method to calculate σ_{tot} is to apply the optical

Table 1. Contributions of various channels to the finite GDH integral (in μb), integrated up to 0.8 GeV for photodisintegration, 1.5 GeV for single pion and η-production and 2.2 GeV for double pion production on nucleon and deuteron, see [37] for further details.

	np	π	$\pi\pi$	η	$\sum$	Sum rule value
neutron		138.95	82.02	−5.77	215.20	233.16
proton		176.38	93.93	−8.77	261.54	204.78
deuteron	−381.52	263.44	159.34	−13.95	27.31	0.65

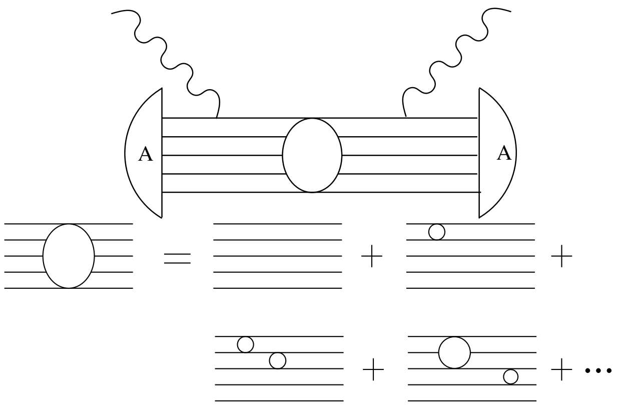

Fig. 15. Top panel: diagrammatic representation of the direct Compton scattering amplitude for an A-nucleon system. Only one-body currents are depicted for the sake of simplicity. Bottom panel: graphical illustration of a selected choice of contributing mechanisms to the imaginary part of the Compton scattering amplitude in (18).

theorem, here to Compton scattering (see fig. 15)

$$\sigma_{tot}(\gamma A \to X; W) \sim \lim_{\epsilon \to 0} \operatorname{Im} T(\gamma A \to \gamma A; W + i\epsilon, \theta = 0) \tag{18}$$

with W as invariant energy of the reaction. In order to obtain the imaginary part, one has to know very precisely the pole structure of the intermediate virtual states between photon absorption and emission which requires a careful numerical treatment of the occurring singularities. It is obvious that with increasing mass number A and increasing energy/momentum transfer this task becomes more and more complicated and finally practically impossible.

An elegant solution of this problem has been proposed about a decade ago by the Trento group [44]. The essential idea is to perform first of all an integral transform of σ_{tot} according to

$$L(\sigma_R, \sigma_I) = \int \mathrm{d}W \frac{\sigma_{tot}(W)}{(W-\sigma)^2} \tag{19}$$

with $\sigma = \sigma_R + i\sigma_I$, where σ_R, σ_I can be treated as free parameters. After some algebra, using the completeness relation of the final states, it turns out that $L(\sigma_R, \sigma_I)$ has the same formal structure as the optical theorem for Compton scattering (18), *i.e.*

$$L(\sigma_R, \sigma_I) \sim \operatorname{Im} T(\gamma A \to \gamma A; \sigma_R + i\sigma_I, \theta = 0). \tag{20}$$

The essential difference between (18) and (20) lies in the argument $W + i\epsilon$ *versus* $\sigma_R + i\sigma_I$. In the optical theorem, the quantity ϵ has to be treated as infinitesimal small yielding in consequence the above-mentioned complicated pole structure. Its counterpart in the LIT, σ_I, is finite and at our disposal. It can, at least in principle, be chosen arbitrarily. This has far reaching consequences, because the pole structure in (19) vanishes for σ_I finite. This yields enormous numerical simplifications, because one needs only *bound state techniques*, avoiding in consequence the calculation of A-body scattering states. In order to obtain the desired inclusive cross section σ_{tot}, one has of course to perform a numerical inversion of the LIT. Recently, a variety of different reliable inversion methods has been presented [45] so that this problem is very well under control. An important cross check for the inversion is that the resulting cross section should be independent of the parameter σ_I so that the LIT method is completely parameter free.

Due to these features, it is not very surprising that the LIT has been applied with considerable success to microscopic calculations of quite a few electroweak cross sections of various nuclei ranging form $A = 2-7$ like inclusive electron scattering (see *e.g.* [46,47]) and total photoabsorption cross sections (see *e.g.* [48,49,50]). In the meantime, it has also been extended to exclusive reactions [51, 52], photopionproduction on the deuteron [53,54] as well as weak processes [55]. This list of applications shows that the LIT approach constitutes an important progress opening up the possibility to carry out *ab initio* microscopic calculations not only for reactions on the classical few-body systems (deuteron, three-body nuclei) but also for reactions on more complex nuclei.

4 Summary and outlook

The study of reactions on few-nucleon systems is of particular importance for testing present theoretical frameworks in terms of effective degrees of freedom. Of specific interest are electromagnetic reactions above pion threshold where a unified approach needs to be constructed. Few-nucleon systems are moreover of importance as effective neutron targets, for example with respect to the extraction of the electric neutron form factor G_{En}. The situation turns out to be much more complicated with respect to the study of the GDH sum rule on the neutron, where — in contrast to G_{En} — no selection of the pure quasi-free kinematics is possible and where many different reaction channels have to be taken into account. Nevertheless, the planned measurements of the GDH spin asymmetry on the deuteron and ^{3}He at MAMI will lead to very stringent tests of our present knowledge of nucleon and nuclear structure. Additional measurements are also desirable for electromagnetic reactions on more complex few-nucleon systems ($A \geq 4$) where nowadays for the first time purely microscopic calculations with the help of the Lorentz integral transform method are possible.

Summarizing, the study of few-nucleon systems is a very active field both from the experimental as well as theoretical point of view. The expected progress will be very important for the future development of hadronic physics in general.

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